**Foundations of Deep Learning – Homework Assignment #2**Adi Album & Tomer Epshtein

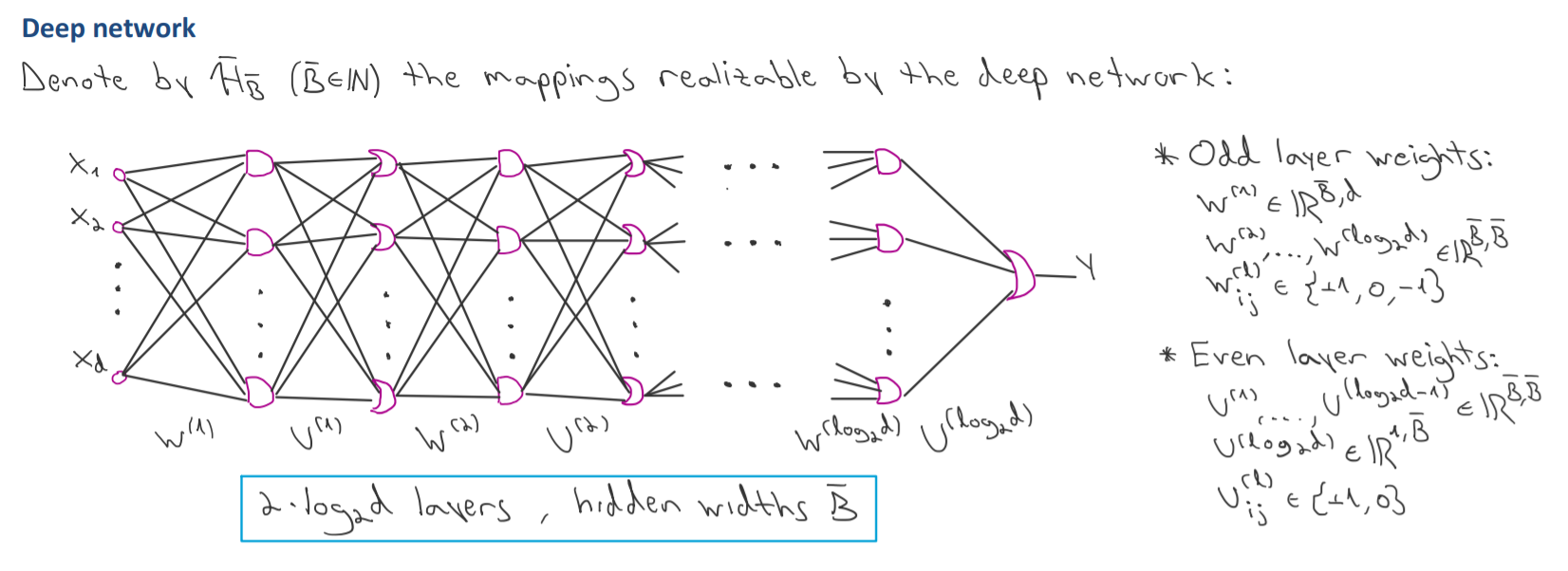
**Part 1: (2)**

Question:  
Prove that with polynomial width the deep network cannot realize all possible functions. In particular derive an exponential in lower bound on required in order for its hypothesis space to be .

Solution:

Denote by the class of functions realizable by deep AND-OR networks with width (as defined in class). We will prove that by combinatorial reasoning for . And that for we need to be exponential in .

Let’s count the number of parameter combinations. Reminder:



* has parameters from , yielding: combinations
* has parameters from , for , yielding: combinations
* has parameters from , yielding: combinations
* has parameters from , yielding: combinations.

Bringing it all together:

Let .

(Where holds because , , and monotonicity of )

For any :  
 and in particular so .  
So,  
for *.*

We will now derive an exponential lower bound for for us to achieve .

We saw

For we must have

.

So for, we must have And in particular .

An exponential in lower bound on , as desired.